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Sodium atoms in Fabry–Perot resonators: studies of static and dynamic behaviour in transverse magnetic fields

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In experiments on a sodium-filled Fabry–Perot resonator optical bistability, tristability and critical slowing-down were observed, as well as magnetically induced self-pulsing (*Phys. Rev. Lett.* **50**, 1660 (1983)). Our experimental results can be explained by the model of Kitano, Yabuzaki & Ogawa (*Phys. Rev.* A **24**, 3156 (1981)) in a qualitative way. In a more detailed treatment we obtain modified equations of motion that make phase space three-dimensional. The results of a stability analysis and of numerical solutions of the equations of motion are compared to experimental observations. We discuss the limitations of the adiabatic elimination of the cavity and demonstrate that not only tristable but also bistable systems can evolve into oscillatory states under the action of a static magnetic field; though the range of allowed experimental parameters is more restricted.

Many experiments on bistability in all-optical systems have been made in Fabry–Perot resonators containing sodium atoms. These experiments are facilitated by the fact that sodium atoms are not at all ideal two-level atoms, but have hyperfine structure and level degeneracy; this introduces hyperfine pumping and Zeeman pumping as very efficient nonlinear mechanisms. We have studied the static and dynamic behaviour of this type of system under conditions of transverse optical pumping.

Under static conditions the non-absorbing resonance related to ground state coherence allows the observation of optical bistability at very low power levels of the laser light source. Both absorptive and dispersive bistability can be obtained. The bistability can be controlled by a static magnetic field (Mlynek *et al.* 1982). Our experimental data can be described in a semiquantitative way by a simple model based on the assumption of three-level atoms (Λ-type scheme) (Mlynek *et al.* 1984).

It should be noted that the nonlinearity produced by Zeeman coherence can be used in related nonlinear optical experiments to advantage. Very recently, for example, multistabilities in intracavity phase conjugation through degenerate four-wave mixing have been observed (Mlynek *et al.* 1983).

We also studied the transient response of the device to step-inputs of light. Our measurements demonstrate the crucial role of critical slowing-down for switching time. We find that time-dependent diffusion processes can strongly affect the dynamics of the bistable device. Numerical calculations yield results in satisfactory agreement with experimental data (Mitschke *et al.* 1983*a*).

While in the experiments mentioned so far the light was always circularly polarized, optical tristability connected with polarization switching is obtained with linearly polarized light. It has been pointed out by Kitano *et al.* (1981) that under the action of the external magnetic field magnetically induced optical self-pulsing should occur, and the phenomenon has been observed recently by Mitschke *et al.* (1983*b*), when the sodium density was sufficiently high. While the basic features of the observations are well explained by the theory of Kitano *et al.* (1981), the following modifications have to be applied to obtain a more quantitative description of the experiment.

(i) In Kitano *et al.*(1981) absorptive losses in the nonlinear Fabry–Perot resonator are not taken into account. On the other hand, the experiment has to be made not too far from resonance to facilitate optical pumping. We find a strong influence of absorption on the pulse shapes. For example, the smoothness and symmetry of the pulses displayed in figure 1 are a consequence of absorption, while the pulse shape displayed in figure 6 of Kitano *et al.* (1981) is typical for the purely dispersive case.

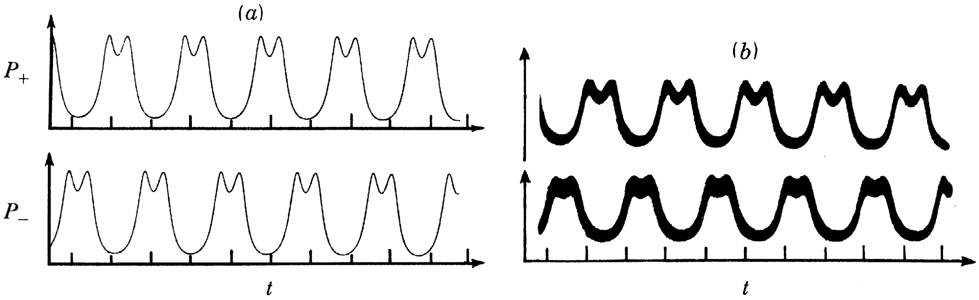

FIGURE 1. Typical time dependence of right (upper trace) and left (lower trace) circularly polarized component of output intensity: (*a*) numerical solution; (*b*) experimental. The scale of the abscissas is 200 ns per division.

(ii) Obviously, adiabatic elimination of the cavity fails when the repetition rate of the pulses is increased by applying a strong magnetic field. We find that the oscillation is quenched by increasing the magnetic field above a limiting value.

(iii) Using the model and abbreviations of Mitschke *et al.* (1983*b*), we find the following equations of motion for the normalized components of magnetization of the sample, which play the crucial role in the explanation of the experiment:

$$\mathrm{d}m_x/\mathrm{d}t = -\Omega_0 m_z - (\Gamma + P_+ + P_-)\, m_x - \Delta(P_+ - P_-)\, m_y, \tag{1a}$$

$$\mathrm{d}m_y/\mathrm{d}t = -(\Gamma + P_+ + P_-)\, m_y + \Delta(P_+ - P_-)\, m_x, \tag{1b}$$

$$\mathrm{d}m_z/\mathrm{d}t = \Omega_0 m_x - (\Gamma + P_+ + P_-)\, m_z + (P_+ - P_-). \tag{1c}$$

Here Ω_0 is the Larmor frequency, P_+ and P_- represent the pumping rates induced by right or left circularly polarized light, respectively, Γ is the relaxation constant of the orientation and Δ is the detuning of the laser with respect to the atomic resonance normalized to the spectral width of the absorption line. The 'dispersive' terms containing Δ are not present in (4*a*) to (4*c*) of Kitano *et al.* (1981), since these are based on an analysis assuming 'broad band' excitation.

The dispersive terms reflect the fact that in a strong off-resonant light field the spins do not simply precess about the magnetic field, but a nutation is superimposed. They strongly influence the shape of the signals, especially in the starting period of the oscillation. We found, for example, that signals like the one in figure 2, where switching occurs via a pronounced oscillatory process, can be described reasonably only if the dispersive terms are taken into account.

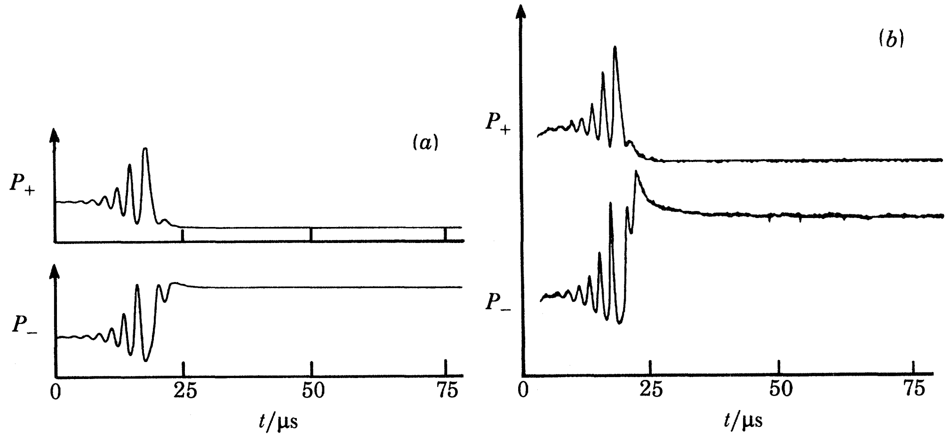

FIGURE 2. Time dependence of output intensity under conditions where the system leaves an unstable focal point and runs into the domain of attraction of a stable fixed point: (*a*) numerical solution; (*b*) experimental.

It should be noted that the dispersive terms make phase space three-dimensional; this might give rise to a much more complicated and possibly chaotic behaviour of the system. There are, however, no indications of chaos in the range of parameters explored in the experiment, neither experimentally nor in numerical solutions of the equations of motion.

Though the concept of magnetically induced optical self-pulsing has been developed in the context of optical tristability, the phenomenon can also occur in bistable systems with circularly polarized light input. In both cases an analytic treatment allowing linear stability analysis can be given and the topology of phase space can be studied in some detail.

REFERENCES

Kitano, M., Yabuzaki, T. & Ogawa, T. 1981 *Phys. Rev.* A **24**, 3156.
Mitschke, F., Deserno, R., Mlynek, J. & Lange, W. 1983*a* *Optics Commun.* **46**, 135.
Mitschke, F., Mlynek, J. & Lange, W. 1983*b* *Phys. Rev. Lett.* **50**, 1660.
Mlynek, J., Mitschke, F., Deserno, R. & Lange, W. 1982 *Appl. Phys.* B **28**, 135.
Mlynek, J., Mitschke, F., Köster, E. & Lange, W. 1983 In *Coherence and quantum optics* V (ed. L. Mandel & E. Wolf), pp. 1179–1186. New York: Plenum Press.
Mlynek, J., Mitschke, F., Deserno, R. & Lange, W. 1984 *Phys. Rev.* A **29**, 1297.